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Stochastic resonance at higher harmonics in monostable systems

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The nonlinear response of a system subject to a periodic force in the presence of noise is investigated. It is shown that a physical system of a general type manifests stochastic resonance at higher harmonics. The stochastic resonance curves are calculated for several monostable systems. Higher harmonic stochastic resonance is measured on a monostable part of a domain wall pinned by a magnetic defect in a ferrite-garnet film. [S1063-651X(97)50611-8]

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Stochastic resonance (SR), which is a noise induced enhancement of the signal-to-noise ratio, has attracted a lot of attention during the last decade [1,2]. Today, SR is recognized as a general principle of the signal-to-noise enhancement in different types of systems. It may play an important role not only in the periodic recurrence of the Earth's ice ages but in the neuron operation and sensory biology [3]. One of the characteristic features of stochastic resonance in systems without a threshold is bistability. Although it is not difficult to find a monostable system where a signal is enhanced by external noise [4], the signal-to-noise ratio generally has a tendency to decrease with a noise increase. Recently it was shown that the signal-to-noise ratio can be enhanced by noise for underdamped single-well systems [5] and for a special type of monostable systems [6]. Other interesting recent examples are given in [7]. On the other hand, new results were obtained in investigations of harmonic mixing [8,9] in stochastic resonance.

In this paper we demonstrate that stochastic resonance at higher harmonics does not require bistability to be observed. It is natural to check mixed harmonics for the stochastic resonance behavior. Indeed, the addition of noise of appropriate intensity pushes a system driven by a small periodic force to the nonlinear region, which leads to an increase of the signals at higher harmonics. If the signal increase is bigger than the increase of the noise component of system motion, the stochastic resonance peak will be observed.

To get the main features of the phenomenon, let us consider an overdamped particle moving in a parabolic potential $U_0(x)=ax^2/2$ perturbed by a small nonlinear correction $u(x)$,

$$\dot{x}=-ax-u'(x)+f_0\cos(\omega_0 t)+\xi(t), \tag{1}$$

where $f_0\cos(\omega_0 t)$ is a small periodic force and $\xi(t)$ is noise with $\langle\xi(\overline{t})\xi(t)\rangle=2D\delta(\overline{t}-t)$.

Writing the solution as

$$x(t)=X_0(t)+X_n(t)+\eta(t), \tag{2}$$

where X_0 is small vibrations produced by the periodic force in the absence of noise and the nonlinearity, X_n is the Ornstein-Uhlenbeck process induced by noise, and $\xi(t)$ is the correction produced by $u(x)$, we get in the first order of $u(x)$:

$$\dot{\eta}=-a\eta-u'(X_0(t)+X_n(t)), \tag{3}$$

and

$$\langle\eta(\overline{t})\eta(t)\rangle=\exp[-a(\overline{t}+t)]\int_{-\infty}^{\overline{t}}\int_{-\infty}^{t}\times\exp[a(\overline{\tau}+\tau)]K(\overline{\tau},\tau)d\overline{\tau}d\tau,$$

$$K(\overline{\tau},\tau)=\langle u'(X_0(\overline{\tau})+X_n(\overline{\tau}))u'(X_0(\tau)+X_n(\tau))\rangle. \tag{4}$$

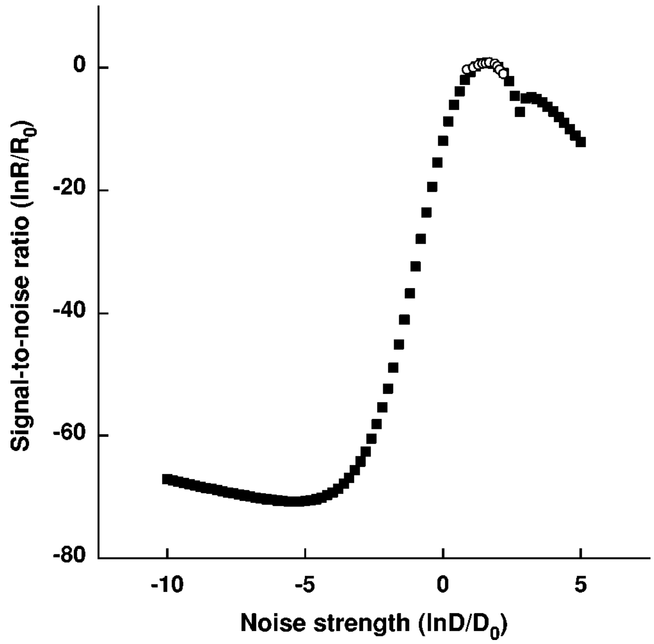

FIG. 1. The signal-to-noise ratio for the parabolic potential $ax^2/2$ with $a=3$ perturbed by a small nonlinearity $u(x) =\epsilon\exp[-(x-x_0)^2/2\delta^2]$ with $x_0=4$, $2\delta^2=0.3$ as a function of the noise strength. Boxes correspond to the exact first order result (6) and open circles are obtained by the computer simulation of the system motion. The frequency of the periodic force used in computer modeling was $\omega_0=0.4$, the force was $f_0=2.0$.

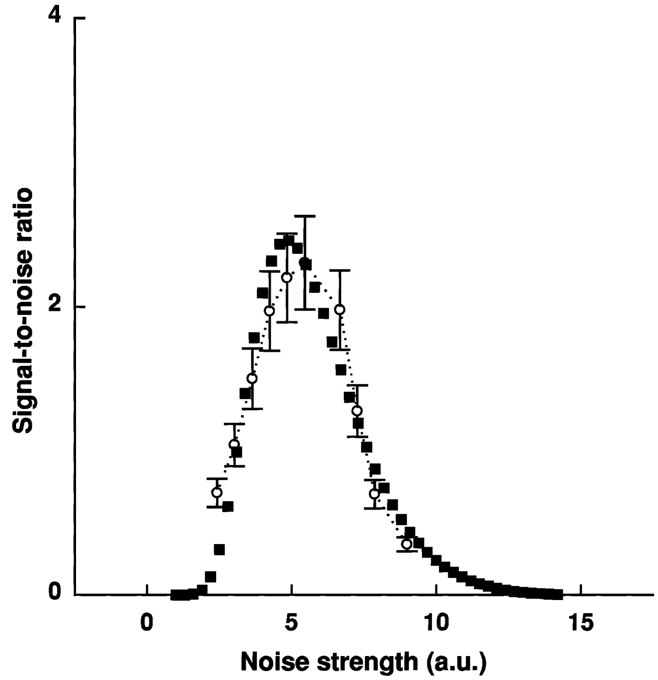

FIG. 2. The signal-to-noise ratio as a function of the noise strength near the stochastic resonance peak. Boxes represent the exact first order result (6) and open circles correspond to the computer simulation. The parameters of the potential and parameters of the simulation are the same as in Fig. 1.

An ensemble averaging yields

$$K(\overline{\tau},\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u'(X_0(\overline{\tau})+\overline{x})\rho_1(\overline{x}-x,\overline{\tau}-\tau) \times u'(X_0(\tau)+x)\rho_0(x)\,d\overline{x}\,dx, \quad (5)$$

where $\rho_0(x)=\sqrt{a/2\pi D}\exp(-ax^2/2D)$ is the stationary distribution and $\rho_1(\overline{x}-x,\overline{\tau}-\tau)$ is the transitional probability of the Ornstein-Uhlenbeck process.

For simplicity we assume that the frequency ω_0 is much smaller than a, so that the main contribution to the integral Eq. (5) comes from the region $\overline{\tau}-\tau\gg 1/a$, where $\rho_1(\overline{x}-x,\overline{\tau}-\tau)\approx\rho_0(\overline{x})$. Then, developing Eq. (5) over X_0 and taking noise in the zero approximation, we get the final result for the signal-to-noise ratio at the nth harmonic ($n\geq 2$):

$$R_{n\omega}=\frac{\pi q_n^2 f_0^{2n}}{2^{n-1}(n!)^2 a^{2n} D\Delta\nu}, \quad (6)$$

where $\Delta\nu$ is the detection frequency bandwidth and

$$q_n=\int_{-\infty}^{\infty}\rho_0(x)u^{(n+1)}(x)dx=\int_{-\infty}^{\infty}\sqrt{\frac{a}{2\pi D}} \times\exp\left(-\frac{ax^2}{2D}\right)u^{(n+1)}(x)dx, \quad (7)$$

where $u^{(n)}(x)$ is the nth spatial derivative. Results for frequency mixing are analogous to Eq. (6). Note that an even potential $u(x)$ $[u(x)=u(-x)]$ produces odd harmonics (R_n disappears for all $n=2m$) and an odd potential $u(x)$ $[u(x) =-u(-x)]$ generates only even harmonics as required by the symmetry arguments.

We can anticipate stochastic resonance at higher harmonics from Eqs. (6), (7). If the nonlinearity is not concentrated at the equilibrium position $x=0$, then the integral (7) will have an effective exponential factor $\exp(-al^2/D)$, where l is an effective length that can be attributed to the potential $u(x)$. This factor together with the factor D in the denominator will lead to the SR peak at mixed harmonics.

To be specific, let us consider the case where the nonlinearity is located at some point x_0:

$$u(x)=\epsilon\exp\left(-\frac{(x-x_0)^2}{2\delta^2}\right). \quad (8)$$

Figure 1 demonstrates the signal-to-noise ratio at the second harmonic calculated with Eqs. (6), (7). The stochastic resonance curve for the potential (8) goes through the minimum $R_{\rm min}$ at $D_{\rm min}\approx a\delta^4/x_0^2$, has a pronounced maximum $R_{\rm max}$ at $D_{\rm max}\approx ax_0^2/9$, and displays a second maximum at $D \approx 4ax_0^2/3$ (it is assumed that x_0/δ is large). A more detailed behavior of the stochastic resonance peak is given in Fig. 2.

It is possible to obtain an exact analytical result for the curve shown in Fig. 1. The straightforward evaluation yields

$$R_{2\omega}=\frac{\pi\varepsilon^2 f_0^4}{8a^5\delta^8\Delta\nu}\,\frac{\lambda\beta^6}{1-\beta}(\beta\lambda-3)^2\exp(-\beta\lambda), \quad (9)$$

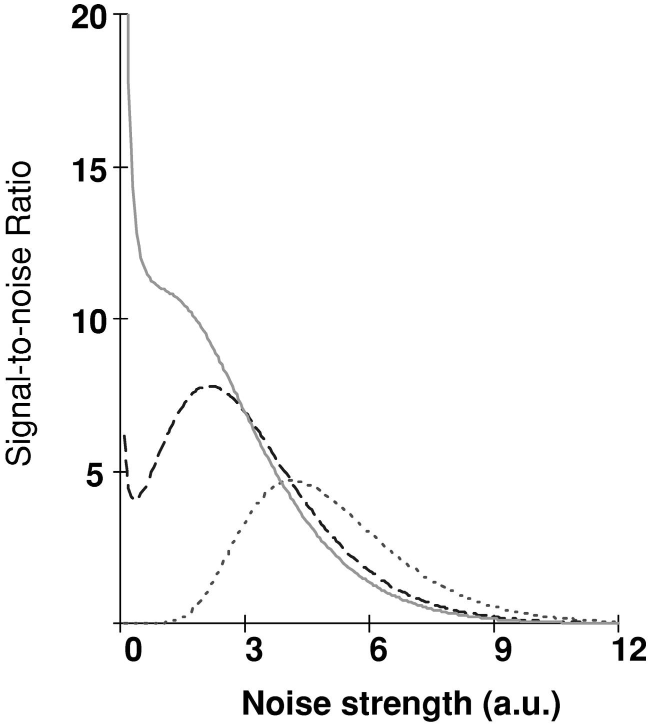

FIG. 3. The signal-to-noise ratio as a function of the noise strength for three different degrees of localization of the nonlinear potential (8): $2\delta^2=2.1$ (solid line), $2\delta^2=1.75$ (dashed line), $2\delta^2=0.7$ (dotted line). The calculation is made using Eqs. (6) and (7) with $a=3$, $x_0=4$, and $\epsilon\sim 1/\delta$ in order to obtain comparable values of the signal-to-noise ratio. The diverging tail of the curve with $2\delta^2=0.7$ at $D\rightarrow 0$ almost coincides with the y axes.

where $\lambda=(x_0/\delta)^2$ and $\beta=a\delta^2/(a\delta^2+D)$. The quality Q of the SR curve, which is defined as the ratio R_{max}/R_{min} [9], for the case $\lambda \gg 1$ is equal to

$$Q=\frac{2^2 3^{14}\exp(\lambda-10)}{(\lambda-1)^6(\lambda-4)^2(\lambda-9)}. \tag{10}$$

The resonance quality Q for the discussed potential depends upon the degree of localization of the nonlinearity (the ratio x_0/δ) and can be arbitrarily large. The resonance peak disappears for $\delta > x_0/4$ where $Q<1$. Figure 3 presents SR curves for three different values of x_0/δ.

The higher harmonic resonance for the nonlinear correction (8) in the noise region $D \gg a\delta^2$ can be approximated as

$$R_{2\omega}=\frac{C}{D^6}\left(\frac{ax_0^2}{D}-3\right)^2\exp\left(-\frac{ax_0^2}{D}\right), \tag{11}$$

where C is a constant. This expression is analogous to the conventional SR formula. Such an analogy was also noted for three main types of SR—the bistable potential model, the fire and reset model, and the simple threshold model [3]. The derivation given above provides a clear physical reason for this analogy. Indeed, signals in SR are generated at some nonzero energy. In the case of Eq. (8) this is the energy $U_0=ax_0^2/2$, where the nonlinearity is located. The probability to find the system at this energy is proportional to $\exp(-U_0/D)$. It results in the factor $\exp(-2U_0/D)$ for the correlation function, which is the product of two system coordinates taken at different times. The preexponential factor arises from the power spectrum of noise and the ability of the system to generate signals.

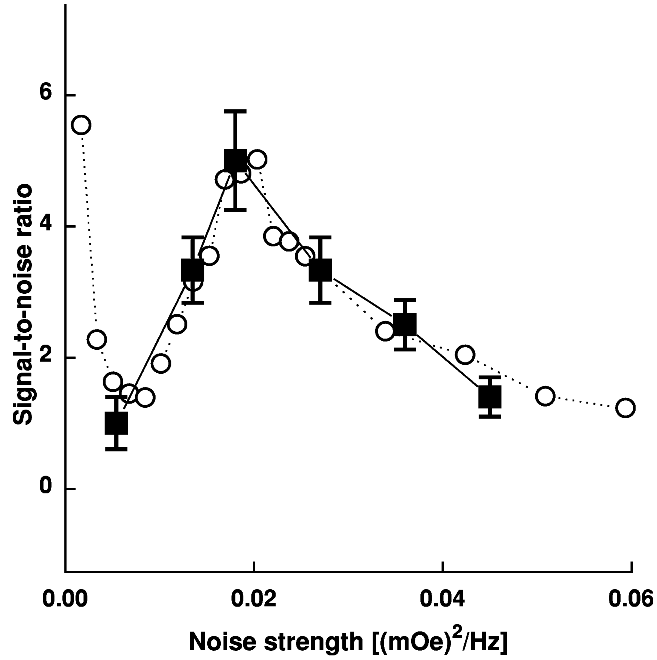

FIG. 4. The signal-to-noise ratio of a local part of a domain wall as a function of the strength of external noiselike magnetic fields: measured values (boxes), and the computer simulation of the system motion in the potential (12) (circles). The parameters of the potential that was used for the modeling are $a=6$, $b=44$, $f_0=1.4$, $\omega_0=3$.

Thus, we have shown that monostable systems demonstrate stochastic resonance at higher harmonics. The most promising is a system with a nonlinearity concentrated at some energy, as Eqs. (6) and (7) indicate. We have checked the conclusion by direct experiments. A local part of an isolated straight domain wall in a thin ferrite-garnet film was studied. An isolated domain wall was produced by an external magnetic field gradient and was placed near a magnetic defect. The domain wall was subjected to an action of periodic and noise-like magnetic fields and its response was measured as a function of the strength of the external random magnetic field. The experimental installation and experimental procedures are described in detail in [9,10].

The domain wall moves in a parabolic defect potential when the amplitude of the driving force is smaller than the defect strength. If the amplitude of the driving field is bigger than the defect strength, the domain wall moves in a parabolic potential of the external gradient field. Thus, all nonlinearity of the total potential of the domain wall is concentrated at the defect energy and we can anticipate that the monostable domain wall demonstrates stochastic resonance at higher harmonics. The measured signal-to-noise ratio at the third harmonic as a function of the noise strength is shown in Fig. 4. (The signal at the second harmonic was zero due to the symmetry of the potential.) In experiments the film of $(LuBi)_3(FeGa)_5O_{12}$ ferrite-garnet was placed in the gradient field of 15 kOe/cm. The film parameters are as follows: thickness: 30 mm; the anisotropy field: 1800 Oe; magnetization: 80 G; the stripe period: 25 mm; the domain wall mobility: 10^3 cm/(Oe sec). The frequency of the driving was 1.1 kHz, the detection frequency was 3.3 kHz, the amplitude of the periodic field was 50 mOe. One can see the stochastic resonance peak at the magnetic field noise strength $D_{\mathrm{mf}}=0.018(\mathrm{mOe})^2/\mathrm{Hz}$ that corresponds to the noise strength $D=\mu^2 D_{\mathrm{mf}}=0.018\ \mathrm{cm}^2/\mathrm{sec}$, where μ is the domain wall mobility. This noise strength should be compared with the defect energy $\mu H_d \delta=0.016\ \mathrm{cm}^2/\mathrm{sec}$, where $H_d=0.4$ Oe is the defect field and $\delta=0.4\ \mu$m is the defect range. The peak position and the resonance quality depend upon the magnitude of the external field gradient.

We have performed a direct computer simulation of the domain wall motion. The monostable potential of a local part of domain wall was chosen in the form

$$U(x)=\frac{ax^2}{2}+\frac{bx^2}{2}\exp\left(-\frac{2x^2}{\delta^2}\right), \tag{12}$$

where a is proportional to the value of the external field gradient, b and δ describe the defect strength and range, respectively. The standard procedure of calculations was used. The results of calculations are in good agreement with measured values; see Fig. 4. A computer simulation was also made for the system moving in the potential (8). The results are represented in Figs. 1 and 2. They are close to the results that were obtained with the help of Eq. (7).

In conclusion, we have shown, both theoretically and experimentally, that the higher harmonic stochastic resonance does not require bistability to be observed. We have calculated the noise induced harmonic mixing for the case of a small nonlinearity and measured the stochastic resonance peak at the third harmonic for a monostable part of an isolated domain wall. The quality of the stochastic resonance at higher harmonics should be high when the nonlinearity of a system is located at some energy. The proposed phenomenon can be applied in situations where conventional stochastic resonance is obliterated by the interference of the driving and measuring circuits, for example, in scanning tunneling microscopy [11].

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